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SPACETIME NONLOCALITY AND RETARDATION IN RELATIVISTIC HEAVY-ION COLLISIONS

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ABSTRACT

We discuss the exact numerical solution of the classical relativistic equations of motion for a Lagrangian corresponding to point nucleons interacting with massive scalar and vector meson fields. The equations of motion contain both external retarded Lorentz forces and radiation-reaction forces; the latter involve nonlocal terms that depend upon the past history of the nucleon in addition to terms analogous to those of classical electrodynamics. The resulting microscopic many-body approach to relativistic heavy-ion collisions is manifestly Lorentz covariant and allows for nonequilibrium phenomena, interactions with correlated clusters of nucleons, and particle production. For point nucleons, the asymptotic behavior of nucleonic motion prior to the collision is exponential, with a range in proper time of approximately 0.5 fm. However, this behavior is altered by the finite nucleon size, whose effect we are currently incorporating into our equations of motion. The spacetime nonlocality and retardation that will be present in the solutions of these equations may be responsible for significant collective effects in relativistic heavy-ion collisions.

1. Introduction

As we discussed at the Sixth Winter Workshop on Nuclear Dynamics,[1] the conditions encountered in relativistic heavy-ion collisions at AGS, CERN, and RHIC energies are very different from those necessary for the valid application of most of the approximation methods and models used previously to describe such collisions. In particular, the interaction time is extremely short, and the nucleon mean free path, force range, and internucleon separation are all comparable in size. To satisfy these conditions *a priori*, a fully microscopic many-body treatment that allows for nonequilibrium phenomena, interactions with correlated clusters of nucleons, and particle production appears necessary. Furthermore, the approach must be manifestly Lorentz covariant, or problems with causality immediately arise. On the other hand, at bombarding energies of many GeV per nucleon, the reduced Compton wavelength of projectile nucleons is sufficiently small that quantal coherence effects are negligible and the classical approximation for nucleon trajectories should be valid. A natural approach that satisfies all these requirements is classical relativistic hadrodynamics, corresponding to nucleons interacting with massive scalar and vector meson fields.

2. Classical Relativistic Hadrodynamics

Our physical input consists of only three axioms: (1) Lorentz invariance, which includes energy and momentum conservation, (2) point nucleons interacting with massive scalar and vector meson fields, and (3) the classical approximation applied in domains where it should be reasonably valid. We then strive for an exact numerical solution of the resulting classical relativistic equations of motion rather than making the usual mean-field approximation or perturbative expansion in coupling strength. For N point nucleons of mass M interacting with scalar and vector meson fields characterized by masses m_s and m_v and coupling constants g_s and g_v, the Lagrangian density is

$$\mathcal{L} = -\frac{1}{2}M\sum_{n=1}^{N}\int d\tau_n\,\delta^{(4)}[x-q_n(\tau_n)]\,v_{n\mu}v_n^{\mu} + \frac{1}{8\pi}\partial_\mu\phi\partial^\mu\phi - \frac{1}{8\pi}m_s{}^2\phi^2 - \rho\phi$$
$$-\frac{1}{16\pi}F_{\mu\nu}F^{\mu\nu} + \frac{1}{8\pi}m_v{}^2A_\mu A^\mu - J_\mu A^\mu \quad . \tag{1}$$

Here ϕ is the scalar potential, ρ is the scalar density, A^μ is the vector potential, $F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$ is the vector field strength tensor, and J^μ is the vector current density. The spacetime trajectory of nucleon n as a function of its proper time τ_n is denoted by $q_n(\tau_n)$, and its four-velocity is $v_n^\mu = dq_n^\mu(\tau_n)/d\tau_n$. We use units in which $\hbar = c = 1$, and a metric specified by $g^{\mu\nu} = \mathrm{diag}(1,-1,-1,-1)$.

The covariant equations of motion for each of the N nucleons that result from the Lagrangian density (1) are[2-4]

$$M^* a^\mu = g_s\,(g^{\mu\nu} - v^\mu v^\nu)\,\partial_\nu\phi_{\rm ext} + g_v F^{\mu\nu}_{\rm ext}v_\nu$$
$$+\left(\frac{1}{3}g_s{}^2 + \frac{2}{3}g_v{}^2\right)\left(\frac{da^\mu}{d\tau} + v^\mu a_\nu a^\nu\right) + f_s^\mu + f_v^\mu \quad , \tag{2}$$

with four-acceleration $a^\mu = dv^\mu/d\tau$ and effective nucleon mass

$$M^* = M + m_s g_s{}^2\int_{-\infty}^{\tau} d\tau'\,\frac{J_1(m_s s)}{s} + g_s\phi_{\rm ext} + \frac{1}{2}m_v g_v{}^2 \quad . \tag{3}$$

The right-hand sides of Eqs. (2) contain both external retarded Lorentz forces and radiation-reaction forces. The latter involve terms analogous to those of classical electrodynamics, as well as the nonlocal terms

$$f_s^\mu = -m_s{}^2 g_s{}^2\,(g^{\mu\nu} - v^\mu v^\nu)\int_{-\infty}^{\tau} d\tau'\,s_\nu\,\frac{J_2(m_s s)}{s^2} \tag{4}$$

and

$$f_v^\mu = m_v{}^2 g_v{}^2 v_\nu\int_{-\infty}^{\tau} d\tau'\,[s^\mu v^\nu(\tau') - s^\nu v^\mu(\tau')]\,\frac{J_2(m_v s)}{s^2} \quad , \tag{5}$$

which depend upon the past history of the nucleon. The four-vector separation is $s^\mu = q^\mu(\tau) - q^\mu(\tau')$, with interval $s = (s_\mu s^\mu)^{1/2}$. Similar equations of motion given in

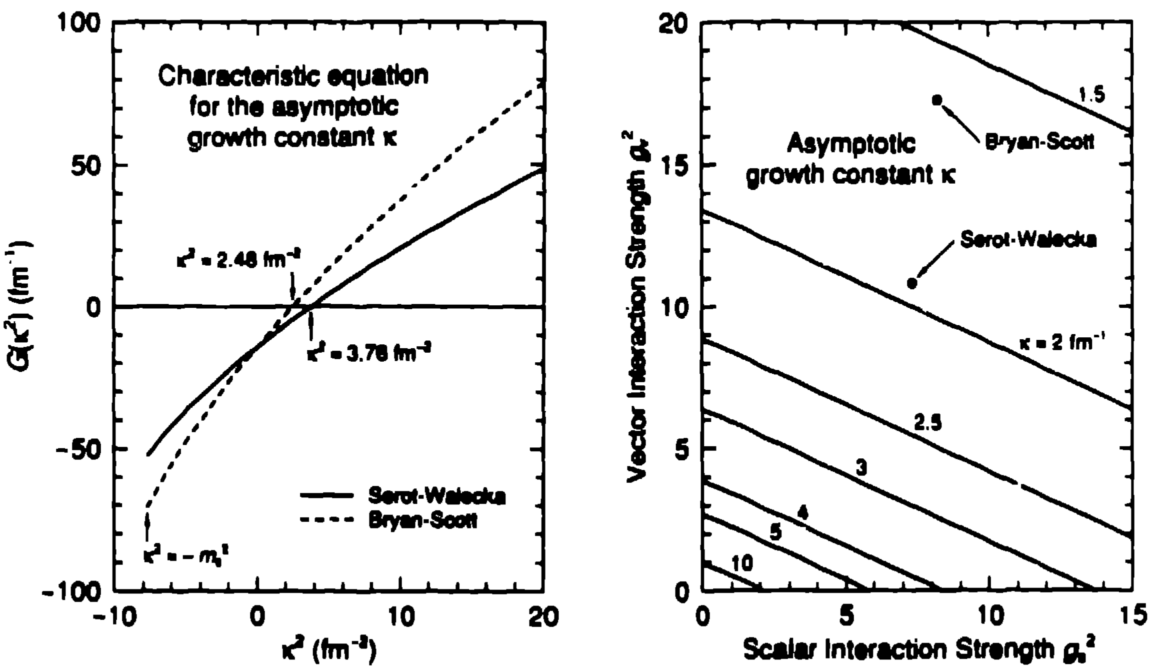

Fig. 1. Plots of the characteristic equation that determines the asymptotic growth constant of point nucleonic motion prior to the collision for two sets of coupling constants (left-hand side), and the resulting dependence of the asymptotic growth constant upon the scalar and vector interaction strengths (right-hand side).

Refs. 1, 2, and 4 for point nucleons interacting with a massive vector field are missing a crucial contribution to the effective nucleon mass, which was responsible for the inability in Ref. 1 to obtain solutions for realistic values of the interaction strength.

We identify the scalar meson field with the σ meson, whose mass we take to be[5] m_s = 550 MeV, and the vector meson field with the ω meson, whose mass is[6] $m_v = 781.95 \pm 0.14$ MeV. For the nucleon mass M we use the average of the neutron and proton masses.

3. Asymptotic Behavior

One can show that an acceleration which grows exponentially, $a^1(\tau) \sim C^1 \exp(\kappa\tau)$, satisfies Eqs. (2)-(5) asymptotically in the limit $\tau \to -\infty$, provided that the asymptotic growth constant κ is determined from the characteristic equation

$$G(\kappa^2) = g_s^2 \left[\left(m_s^2 + \kappa^2\right)^{3/2} - m_s^3 - \frac{3}{2} m_s \kappa^2 \right] / \kappa^2$$
$$+ g_v^2 \left[m_v^3 - \left(m_v^2 - 2\kappa^2\right) \left(m_v^2 + \kappa^2\right)^{1/2} - \frac{3}{2} m_v \kappa^2 \right] / \kappa^2 - 3M = 0 \quad . \quad (6)$$

This equation is plotted in the left-hand side of Fig. 1 for two sets of coupling constants. The first set corresponds to the values g_s^2 = 7.29 and g_v^2 = 10.81 that are

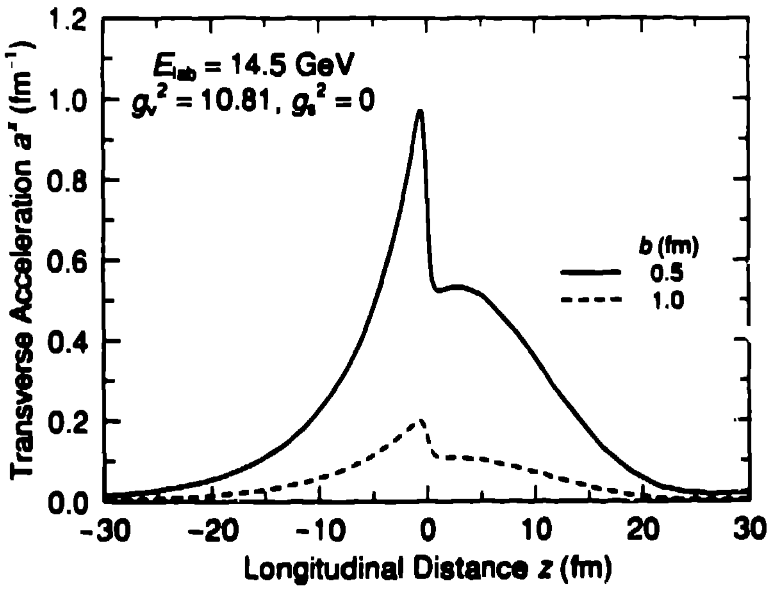

Fig. 2. Transverse four-acceleration a^x experienced by point nucleons, for two values of the impact parameter b.

required to properly saturate symmetric nuclear matter in the mean-field treatment of the relativistic σ-ω model of Serot and Walecka.[5] The second set corresponds to the values $g_s{}^2 = 8.19$ and $g_v{}^2 = 17.26$ determined by Bryan and Scott[7] from an analysis of nucleon-nucleon scattering at laboratory kinetic energies between 0 and 350 MeV.

The inverse of κ gives the range in proper time of the spacetime nonlocality that is present for point nucleons. This nonlocality range is $\kappa^{-1} = 0.516$ fm for the Serot-Walecka constants and $\kappa^{-1} = 0.635$ fm for the Bryan-Scott constants. As shown in the right-hand side of Fig. 1, the asymptotic growth constant κ decreases and the nonlocality range κ^{-1} increases with increasing scalar or vector interaction strength.

4. Numerical Solution

For some special cases, we have solved Eqs. (2)–(5) by converting the differential equations into the integral equations

$$a^\mu(\tau) = \frac{1}{g^2}\int_\tau^\infty d\tau' \left[g_s(g^{\mu\nu} - v^\mu v^\nu)\partial_\nu\phi_{ext} + g_v F^{\mu\nu}_{ext}v_\nu + f^\mu_s + f^\mu_v + g^2 v^\mu a_\nu a^\nu\right]$$
$$\times \exp\left(-\int_\tau^{\tau'} d\tau'' M^*/g^2\right) \quad , \tag{7}$$

with $g^2 = \frac{1}{3}g_s{}^2 + \frac{4}{3}g_v{}^2$. These equations, which are solved iteratively by use of a finite-difference method, automatically satisfy the boundary conditions[8] that $a^\mu(\tau) \to 0$ as $\tau \to +\infty$. Figures 2 and 3 are examples for a single point nucleon incident at 14.5 GeV kinetic energy on a scattering center that remains fixed in the laboratory

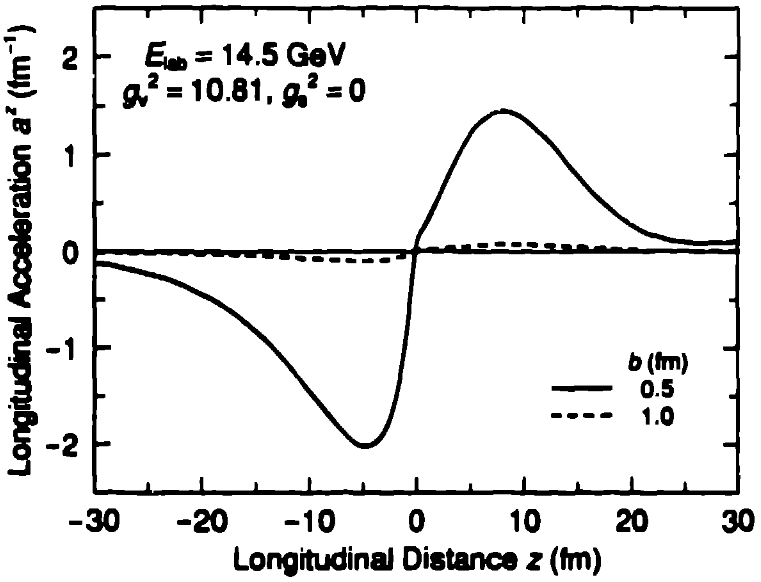

Fig. 3. Longitudinal four-acceleration a^z experienced by point nucleons, for two values of the impact parameter b.

system. These examples are calculated for a vector interaction only, although we have subsequently obtained solutions for both scalar and vector interactions simultaneously. Because of the spacetime nonlocality, which is magnified in a given frame by the Lorentz factor γ, the collision process extends over an appreciable region of spacetime.

5. Effect of the Finite Nucleon Size

In the nonrelativistic limit, the validity of the above classical treatment of point nucleons requires that the spacetime nonlocality range κ^{-1} be large compared to both the quantal uncertainty and the size of the nucleon. The former requirement is actually satisfied in the nuclear case, even though it is violated in classical electrodynamics.[9] The difference arises because the nucleon mass is about 1800 times the electron mass and the strong interaction strength is about 1300 times the electromagnetic interaction strength. However, the nonlocality range of approximately 0.5 fm in the nuclear case is comparable to the size of the nucleon. In particular, the proton charge distribution is approximately exponential in form, with a root-mean-square radius[10] of 0.862 ± 0.012 fm. We are currently incorporating the effect of the finite nucleon size into our equations of motion. For an extended charged particle in nonrelativistic classical electrodynamics, the exponential asymptotic behavior is removed when the size of the particle exceeds the nonlocality range, with the spacetime nonlocality shifted to the region following the collision.[11] We anticipate a similar dependence on size in the nuclear case.

6. Conclusions

Classical relativistic hadrodynamics, corresponding to nucleons interacting with massive scalar and vector meson fields, permits the systematic study of many-body collective dynamics in a way that is manifestly Lorentz covariant. Spacetime nonlocality, retardation, nonequilibrium phenomena, interactions with correlated clusters of nucleons, and particle production are all included automatically. However, it is crucial in this approach to incorporate the effect of the finite nucleon size into the equations of motion. The spacetime nonlocality and retardation that will be present in the solutions of these equations may be responsible for significant collective effects in relativistic heavy-ion collisions.

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